Economic Modelling for Groundwater Resources Management

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- Historically, surface water has been the main source of water for human consumption, as it was easy and cost effective to access.
- However, increased rainfall shortages, especially in areas such as the Mediterranean islands, have resulted in increased use of groundwater
- Demand: domestic, agricultural and environmental/ecosystem preservation.
- Groundwater withdrawals now constitute one third of the world's freshwater consumption.
- This extensive use has resulted in :
  - depletion of groundwater resources
  - biodiversity loss due to adverse effects on wetlands
  - pollution of groundwater resources because of percolation of pollutants associated with agricultural activities
  - seawater intrusion in coastal aquifers

- The needs for groundwater resources along with the acute scarcity of groundwater in many parts of the world, gives rise to the necessity of making choices about how this resource should be allocated among competing uses and over time.
- This is a very interesting management problem.
- The purpose of the present paper is:
- to present some conceptual models related to the management of groundwater resources
- to compare socially-optimal and privately-optimal, or atomistic, management rules and outcomes using game theoretic solutions
- to explore decentralized policy schemes, in the form of water taxes, that can be used to sustain socially-optimal use of groundwater resources.

Groundwater Management: Socially-Optimal and Game Theoretic Solutions

Let $S(t)$ denote the groundwater stock level at time t, Let $x_i(t)$ represent groundwater extraction by agent $i=1,\dots,n,$ and let $\mathbf{x}(t)=(x_1(t),\dots,x_n(t))$ be the vector of extractions at time $t.$ An agent could be, for example, a farmer or any other decision-making unit that can extract water from the aquifer.

The aquifer's stock evolves as:

$$\frac{dS}{dt}=R(S)-x\,,\; x=\sum_{i=1}^{n}x_i\,,\; S(0)=S^0$$

Let $y_i(x_i(t))$ denote benefits (e.g. agricultural production) accruing to economic agent $i,$ by extracting $x_i(t)$ water at time $t.$ Then total benefits are defined as

$$Y(t)=\sum_{i=1}^{n}y_i(x_i(t))\;,y_i^{'}(x_i)>0,\; y_i^{''}(x_i)<0$$

The total cost of extracted water by agent i when the stock is S is given by $C(S)x_i$, where the common unit cost $C(S)$ is nonincreasing and convex.

The socially-optimal solution

Assume that a water authority manages the aquifer. The aim of the authority is to choose time paths for water extraction which will then be assigned to the individual agents, such that total benefits accruing from the use of the aquifer's water are maximized. This is a formal optimal control problem that determines a socially-optimal solution for the aquifer, and which can be stated as

$$\max_{\{\mathbf{x}(t)\}} \int_0^\infty e^{-\rho t} \sum_{i=1}^{n} \left[y_i(x_i(t)) - C(S) x_i(t) \right] dt$$

subject to

$$\frac{dS}{dt} = \dot{S} = R(S) - x, \; x = \sum_{i=1}^{n} x_i, \; S(0) = S^0$$

The optimality conditions derived by the maximum principle imply that

x_i^* maximizes the Hamiltonian function

or $y_i'\left(x_i^*\right) = C(S) + \lambda \Rightarrow x_i^* = x_i^*(S,\lambda)$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial x} = \left(\rho - R'(S)\right)\lambda + C'(S)\sum_{i=1}^{n} x_i^*(S,\lambda)$$

$$\dot{S} = R(S) - \sum_{i=1}^{n} x_i^*(S,\lambda)$$

Solution of the system of differential equations will determine the socially-optimal time paths $\left(S^*(t), \lambda^*(t)\right)$ and the socially-optimal steady-state equilibrium $\left(S^\infty, \lambda^\infty\right)$, defined as the limit of $\left(S^*(t), \lambda^*(t)\right)$ as $t \to \infty$, for the water stock S and its shadow value λ, as well as the corresponding socially-optimal extraction paths $x_i^*\left(S^*(t), \lambda^*(t)\right)$.

If we consider that extractions decisions are taken individually where each economic agent has open access to the aquifer, that is, the aquifer is a common pool resource, there are three possible behavioural rules:

Atomistic solutions

1) **Myopic equilibrium:** The economic agent maximizes current profits and treats the groundwater stock level as fixed at a level $\bar{S}$ without taking into account the evolution of the water stock defined by (stock1). This *myopic* extraction rule determines extraction as:

$$x_i^0(t) : y_i'\left(x_i^0\right) = C(\bar{S})$$

It is clear that by ignoring groundwater scarcity rents, extraction is higher than the socially-optimal, $x_i^0(t) > x_i^*(t)$ and the resource tends to be depleted faster. Since this is basically an open access resource harvesting problem, this solution indicates tragedy of the commons.

Open Loop Nash Equilibrium: The economic agent takes into account the evolution of the water stock, but maximizes the present values of his/her net benefits, by choosing his/her extraction path and by treating the extractions paths of the other agents as fixed at a best response level. The problem can be set up as an n player noncooperative differential game, where extraction paths $\{x_i(t)\}$ are each agent's strategies. The strategy space is determined by the information structure of the game. In an *open loop* information structure individual extractions are defined as

$$\mathrm{OL}: x_i(t) = h_i\left(S^0, t\right) \quad i = 1, \ldots, n$$

Since each agent's strategy depends only on the initial water stock S^0, the problem can be written as

$$\max_{\{\mathbf{x}(t)\}} \int_0^\infty e^{-\rho t}\left[y_i(x_i(t)) - C(S)x_i(t)\right]dt$$

subject to

$$\frac{dS}{dt} = \dot{S} = R(S) - x_i - \sum_{j \neq i}^{n} \bar{x}_j \,, \; S(0) = S^0, \bar{x}_j = h_i\left(S^0, t\right)$$

The solution of the problem (OL1)-(OL2) corresponds to an *Open Loop Nash Equilibrium* (OLNE)

By comparing the socially-optimal solution with the OLNE it is clear that $\lambda^{OL} < \lambda$ so that the OLNE values resource stocks less than the social optimum and therefore extraction is higher, or $x_i^{*OL}(t) > x_i^{*}(t)$. It also holds that $x_i^{0}(t) > x_i^{*OL}(t) > x_i^{*}(t)$. This is because the individual extraction effects on costs are partially internalized in the OLNE, through the term $C^{'}(S) x_i^{*OL}\left(S, \lambda_i^{OL}\right)$ in, so that extractions are less than the myopic rule. But internalization is not full as in the social optimum, where full internalization is obtained through the term $C^{'}(S) \sum_{i=1}^{n} x_i^{*}(S, \lambda)$.

Feedback Nash Equilibrium: Under a feedback (FB) information structure the strategy depends on the current state of the system, that is the current water stock $S(t)$ and time. Therefore with an FB information structure individual extractions are defined as:

$$\text{FB}: x_i^{FB}(t) = h_i(S(t), t) \quad i = 1, \ldots, n$$

The FB strategy described by (FB) is often referred to as a *Markov perfect strategy* in which the water stock is a `sufficient statistic' for the history of the game.

Under the FB information structure the problem can be written as:

$$\max_{\{\mathbf{x}(t)\}} \int_0^{\infty} e^{-\rho t} \left[y_i(x_i(t)) - C(S) x_i(t) \right] dt$$

subject to

$$\frac{dS}{dt} = \dot{S} = R(S) - x_i - \sum_{j \neq ij}^{n} h_i(S(t)) \;,\; S(0) = S^0$$

The ranking of extraction paths and steady state water stock is:

$$x_i^0(t) > x_i^{*FB}(t) > x_i^{*OL}(t) > x_i^*(t)$$

$$S^0 < S^{\infty FB} < S^{\infty OL} < S^{\infty}$$

Thus management under the water authority leads to greater water conservation relative to atomistic equilibria.

Regulation

The water authority can regulate the system by using as decentralized instruments water taxes or water quotas.

Water quotas can be defined as a path of quotas such that individual water extractions do not exceed $x_i^*(t)$.

Water taxes depends on what assumptions are made about the behaviour of individual agents. The target of regulation through water taxes is to make the agents conditions for determining the optimal amount of water used equal to the socially-optimal condition.

Under the tax scheme individual benefits are defined as

$y_i(x_i(t)) - C(S)x_i(t) - \tau(t)x_i(t)$, where $\tau(t)$ is a time flexible water tax. Thus water taxes can be defined as:

$$\text{Myopic Behavior}: \tau_1(t) = \lambda(t)$$

$$\text{OL Behavior}: \tau_2(t) = \lambda(t) - \lambda^{OL}(t)$$

$$\text{FB Behavior}: \tau_3(t) = \lambda(t) - \lambda^{FB}(t)$$

Steady State taxes

$$\text{Myopic Behavior}: \tau_1 = \lambda^\infty$$

$$\text{OL Behavior}: \tau_2 = \lambda^\infty - \lambda^{\infty OL}(t)$$

$$\text{FB Behavior}: \tau_3 = \lambda^\infty - \lambda^{\infty FB}$$

A numerical example

To illustrate the above ideas we proceed with a numerical example.
Let

$$y(x_i) = \ln x_i$$
$$C(S) = 1/S$$
$$R(S) = F - bS$$

$F = 100,\ b = 0.1\ ,\quad \rho = 0.01, \quad n = 10$

At the **social optimum** we obtain the steady-state levels of the water stock, its shadow value, and steady-state extractions as

$$S^{\infty} = 91{,}3\ \lambda^{\infty} = 0.099,\ x_i^{\infty} = \frac{1}{\left(1/S^{\infty} + \lambda^{\infty}\right)}$$

This steady state is a local saddle point since the eigenvalues of the corresponding modified Hamiltonian dynamic system (MHDS), are $(0.483528, -0.473528)$.

Solution **for the OLNE** provides a steady state

$$S^{\infty OL} = 34.3{,}3\ \lambda^{\infty OL} = 0.074,\ x_i^{\infty OL} = \frac{1}{\left(1/S^{\infty OL} + \lambda^{\infty OL}\right)}$$

which has also the local saddle point property since the eigenvalues are $(-1.17155, 0.469934)$. It is clear that the OLNE leads to overexploitation of the aquifer relative to the social optimum.

The **steady-state taxes** are defined as:

$$\tau_1 = 0.099$$

$$\tau_2 = 0.099 - 0.074 = 0.025$$

The **optimal policy function** determines the optimal water extraction $x_i^*(t)$ for each value of the water stock $S(t)$. Thus the policy function can be written as

$x_i^*(t) = \phi(S(t))$,

The linearized policy function as

$$x_i^*(t) = \frac{1}{1/S(t) + \lambda^{\infty} + \gamma(S(t) - S^{\infty})}, \gamma = -0.000332404$$

A water quota x_i^q can then be set so that $x_i^q(t) \leq x_i^*(t)$.

Groundwater Management: Quantity-Quality Problems

In a general quantity-quality (q-q) problem, the deterioration of the quality of a resource due to pollution results in the reduction of the effective use of the resource. Thus the management of a resource should account for both its use and the emission of the pollutants that influence the effectiveness of its use. In the case of groundwater management, water which is pumped by agents (farmers) from a common access aquifer for irrigation purposes, results in deep percolation that causes accumulation of pollutants in the aquifer. Pollution negatively affects the production of the agricultural output through the deterioration of the irrigation water quality.

A q-q problem can be set as follows. Let P be the stock of pollutants (e.g. salinity) accumulated in an aquifer. The stock of pollutants is a negative externality in the production process. Thus the benefit function can be written as:

$$y_i(t) = y_i(x_i(t), P(t)), \frac{\partial y_i}{\partial P} < 0, \frac{\partial^2 y_i}{\partial P^2} < 0, \frac{\partial^2 y_i}{\partial P \partial x_i} \leq 0$$

The evolution of the pollution stock is given by

$$\dot{P} = g(x(t)) - bP, P(0) = P^0, \; x(t) = \sum_{i=1}^{n} x_i(t)$$

In this model $g(x(t))$ is an increasing convex function which can be regarded as reflecting an emission function associated with pollution accumulation, while $b \geq 0$ reflects the aquifer's self cleaning capacity.

In this q-q problem, the social optimum is defined as the solution of the following problem:

$$\max_{\{\mathbf{x}(t)\}} \int_0^\infty e^{-\rho t} \sum_{i=1}^{n} \left[y_i(x_i(t), P(t)) - C(S) x_i(t) \right] dt$$

subject to

$$\frac{dS}{dt} = \dot{S} = R(S) - x, \quad x = \sum_{i=1}^{n} x_i, \quad S(0) = S^0$$

$$\dot{P} = g(x(t)) - bP, \quad P(0) = P^0$$

Regulation

The time flexible water taxes are defined, under symmetry, as:

$$\text{Myopic Behavior}: \tau_1(t) = \lambda(t) + \mu(t)\frac{\partial g}{\partial x_i}$$

$$\text{OL Behavior}: \tau_2(t) = \left(\lambda(t) - \lambda^{OL}(t)\right) + \left(\mu(t) - \mu^{OL}(t)\right)\frac{\partial g}{\partial x_i}$$

$$\text{FB Behavior}: \tau_3(t) = \left(\lambda(t) - \lambda^{FB}(t)\right) + \left(\mu(t) - \mu^{FB}(t)\right)\frac{\partial g}{\partial x_i}$$

Furthermore, the corresponding steady-state taxes are:

$$\text{Myopic Behavior}: \tau_1 = \lambda^{\infty} + \mu^{\infty}\frac{\partial g}{\partial x_i}$$

$$\text{OL Behavior}: \tau_2 = \left(\lambda^{\infty} - \lambda^{\infty OL}\right) + \left(\mu^{\infty} - \mu^{\infty OL}\right)\frac{\partial g}{\partial x_i}$$

$$\text{FB Behavior}: \tau_3 = \left(\lambda^{\infty} - \lambda^{\infty FB}\right) + \left(\mu^{\infty} - \mu^{\infty FB}\right)\frac{\partial g}{\partial x_i}$$

A Numerical Example

We assume that damages due to the pollution externality are given by a damage function $(1/2)P^2$, so net benefits are defined as

$$\ln x_i - (1/S)x_i - (1/2)P^2$$

We assume that the stock of pollution in the aquifer evolves according to

$$\dot{P} = v(nx_i) - \delta P, \; x_i \text{ the same for all } i$$

where v is a fixed unit emission coefficient and $\delta \geq 0$ reflects the aquifer's self cleaning capacity.

Assuming furthermore that $v = 0.2$ and $\delta = 0.05$ we obtain the steady-state levels of the water stock, its shadow value, the pollution stock and its shadow cost, and steady-state extractions as

$$S^\infty = 997.262, \; \lambda^\infty = 0.00907257,$$

$$P^\infty = 1.09529, \mu^\infty = -182.549, x_i^\infty = \frac{1}{(1/S^\infty + \lambda^\infty - v\mu^\infty)}$$

This steady state is a local saddle point Thus a two-dimensional stable manifold exists, and for any initial values for S and P in the neighborhood of the steady state, initial values for λ and μ and consequently x_i can be chosen so that the system converges to the steady state.

Solution for the OLNE provides a steady state

$$S^{\infty OL} = 991.341 \; \lambda^{\infty OL} = 0.002578,$$

$$P^{\infty OL} = 3.46356, \; \mu^{\infty} = -57.7261$$

$$x_i^{\infty} = \frac{1}{\left(1/S^{\infty OL} + \lambda^{\infty OL} - v\mu^{\infty OL}\right)}$$

Social Optimum

$$S^{\infty} = 997.262, \;\; \lambda^{\infty} = 0.00907257,$$

$$P^{\infty} = 1.09529, \; \mu^{\infty} = -182.549, \; x_i^{\infty} = \frac{1}{\left(1/S^{\infty} + \lambda^{\infty} - v\mu^{\infty}\right)}$$

The steady-state taxes are defined as:

$$\tau_1 = 0.00907257 + (0.2)(182.549)$$

$$\tau_2 = (0.00907257 - 0.002578) + (0.2)(-182.549 + 57.7261)$$

It is clear that the water tax is higher relative to the no pollution case since it is adjusted to take into account the production externality.

Concluding Remarks

- The purpose of this paper was to present models related to groundwater management. Two models were developed: one in which only the quantity of the water in an aquifer was the management objective, and a second in which both the quantity and the quality of the aquifer's water were managed.
- In each model alternative management regimes were examined: the socially-optimal management problem, where the purpose was to manage water and pollution stock by maximizing total benefits in a given region, and the non cooperative or atomistic problems, where the objective of each individual agent was to maximize own benefits.
- We define decentralized water taxes both for the pure quantity and the quantity-quality problem capable of regulating the system so that the socially-optimal outcome is achieved. Numerical estimates confirming the theoretical analysis were obtained for the cases of myopic and open loop equilibrium.
- The complete characterization of feedback solutions and the corresponding water taxes for general problems is a current open research problem .